Interference of Light

Huygen's wave theory of light.

- Transverse wave
- construction of wave front

Huygen's principle:

- Secondary wavelets

Light wave - e-m wave $E = E_0 \cos(\omega t - kz) = E_0 e^{-i(\omega t - kz)}$

In general $E = E_0 e^{i(\omega t - \vec{k}\cdot\vec{r})}$ $k = \frac{2\pi}{\lambda}$ = wave constant

- monochromatic wave and achromatic
- Phase difference and coherence
- Path difference

Principle of superposition: Enunciated by Young in 1801.

$y = y_1 + y_2 + y_3 + \cdots$

Interference

Conditions

- should move in same direction.
- same frequencies and wavelengths
- same or nearly same amplitudes.
- should be narrow and close.
- should be coherent
- should be in same state of polarisation.

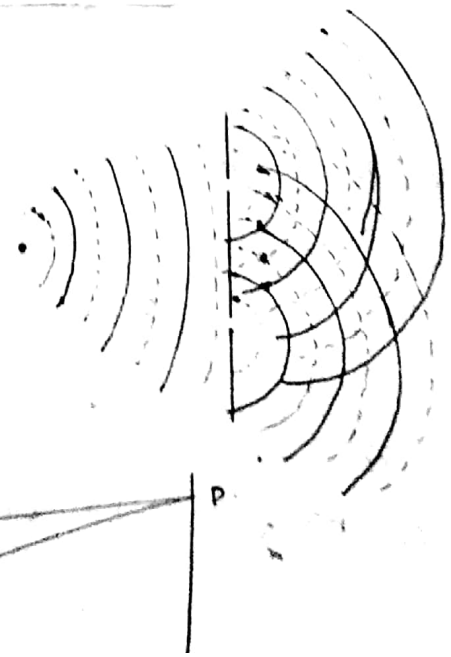
Effect: Intensity is redistributed. Intensity changes with respect to position.

Division of amplitude and wavefront.

- Division of wavefront:

Young's double slit experiment

- constructive interference
- Destructive interference



Analytical treatment of Interference

$y_1 = a e^{-i(\omega t - kx_1)}$

$y_2 = a e^{-i(\omega t - kx_2)}$

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Resultant wave

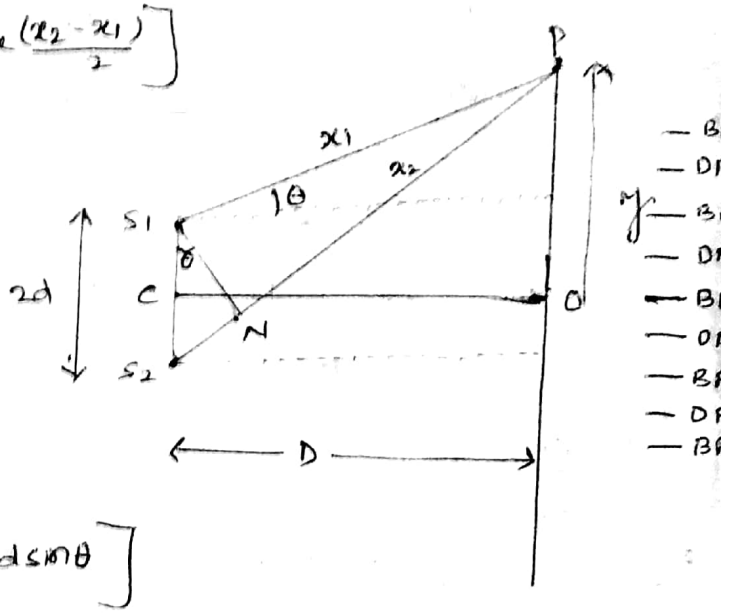
$y = y_1 + y_2 = a e^{-i\omega t}\left[e^{ikx_1} + e^{ikx_2}\right]$

x_1 = Path of 1st ray, x_2 = Path of 2nd ray, $x = \frac{x_1 + x_2}{2}$ = Mean path

$$y = a e^{-i\omega t} e^{ikx}\left[e^{ikx_1} e^{-ikx} + e^{ikx_2} e^{-ikx}\right]$$

$$= a e^{-i(\omega t - kx)}\left[e^{ik(x_1 - x)} + e^{ik(x_2 - x)}\right]$$

$$= a e^{-i(\omega t - kx)}\left[e^{ik\left(\frac{x_1 - x_2}{2}\right)} + e^{ik\left(\frac{x_2 - x_1}{2}\right)}\right]$$



$x_2 - x_1$ = Path difference $= S_2N$

$= 2d\sin\theta = \Delta$

∴ Phase diff = $\frac{2\pi}{\lambda} \times$ Path difference

$\delta = \frac{2\pi}{\lambda} 2d\sin\theta = k2d\sin\theta$

$$\therefore y = a e^{-i(\omega t - kx)}\left[e^{ikd\sin\theta} + e^{-ikd\sin\theta}\right]$$

$$= 2a\cos(kd\sin\theta)\, e^{-i(\omega t - kx)}$$

$$\boxed{y = A e^{-i(\omega t - kx)}}$$

Resultant wave at P whose amplitude is $A\left[= 2a\cos(kd\sin\theta)\right]$

∴ Resultant intensity

$I \propto 4a^2\cos^2(kd\sin\theta)$

$\propto A a^2 \cos^2\left(\frac{\delta}{2}\right)$

∴ $I = 4Ka^2\cos^2(\delta/2)$, K = constant.

Constructive interference.

$I_{max} = 4Ka^2$ when $\cos^2(\delta/2) = +1$

∴ or $\cos(\delta/2) = \pm 1 = \cos n\pi$

∴ $\delta = 2n\pi$ or $\Delta = n\lambda$ where $n = 0, 1, 2, 3 \ldots$

∴ If phase difference $= 2n\pi$ or path difference $= n\lambda$, constructive interference occurs and intensity is maximum.

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Destructive interference

$I = 0$ when $\cos^2(\delta/2) = 0 = \cos(\delta/2) = \cos\frac{(2n+1)\pi}{2}$

$\therefore \delta = (2n+1)\pi$

or $\Delta = (2n+1)\lambda/2$

Fringe width

$S_1P^2 = D^2 + (y-d)^2$, $S_2P^2 = D^2 + (y+d)^2$

$S_2P^2 - S_1P^2 = 4yd$.

$S_2P - S_1P = \frac{4yd}{S_2P + S_1P} = \frac{4yd}{2D} = \frac{2yd}{D}$

For constructive interference.

$\frac{2y_n d}{D} = n\lambda \Rightarrow y_n = n\lambda\left(\frac{D}{2d}\right)$

For destructive interference.

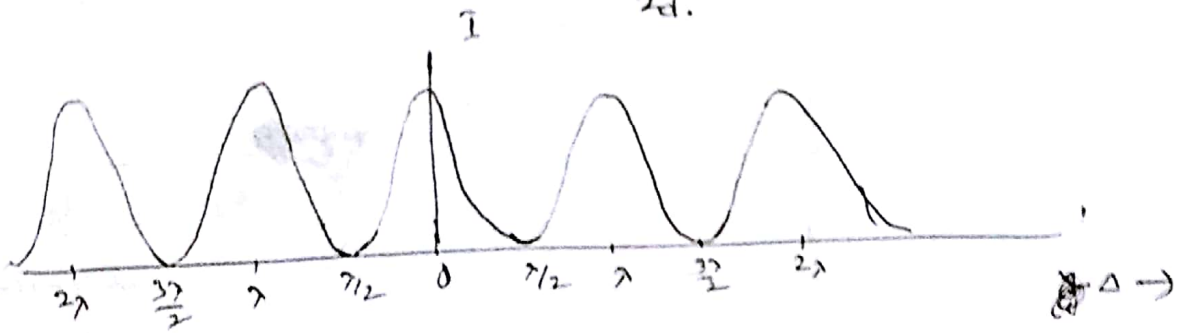
$\frac{2y_n d}{D} = (2n+1)\frac{\lambda}{2} \Rightarrow y_n = \frac{(2n+1)\lambda}{2}\left(\frac{D}{2d}\right)$

$\therefore \bar{\beta} = y_{n+1} - y_n = (n+1)\lambda\left(\frac{D}{2d}\right) - n\lambda\left(\frac{D}{2d}\right) = \frac{\lambda D}{2d}$

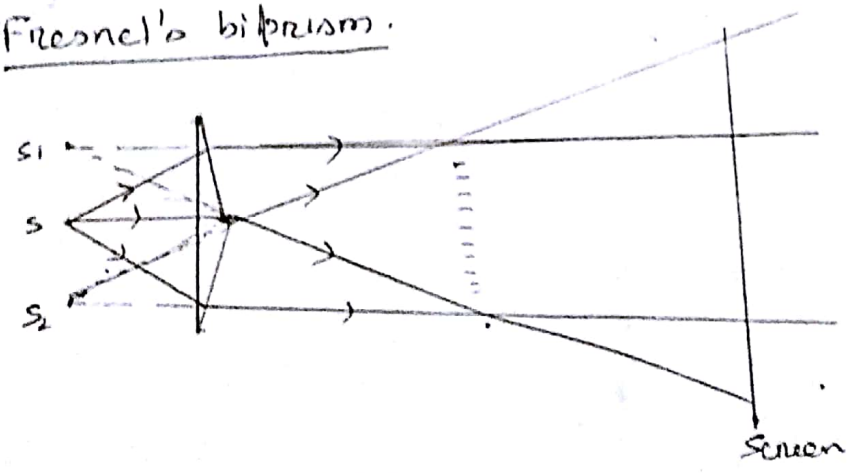
$\bar{\beta} = \frac{\lambda D}{2d}$. , $\bar{\beta} \propto \lambda$

$\propto D$

$\propto \frac{1}{2d}$.



Fresnel's biprism.



$d = \sqrt{d_1 d_2}$, Two conjugate points of lens.

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